ANALYTICAL RELATIONSHIP FOR DETERMINING THE RELAXATION LOSSES OF SPECIFIC MOMENTUM [IN A SUPERSONIC NOZZLE]

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An approximate equation is proposed for determining losses of specific momentum in a supersonic nozzle due to the lack of chemical equilibrium in the flow.

The changes taking place in the composition of the combustion products in supersonic jet flow lag relative to the composition corresponding to chemical equilibrium; the specific momentum accordingly lies below its ideal value. In the course of outflow from the nozzle other relaxation phenomena also occur; however, on using a homogeneous working substance the effect of these on the specific momentum is slight and is usually not taken into account in practical calculations.

By the loss coefficient of specific momentum due tothe lack of chemical equilibrium in the flow (ξ_n) we mean the ratio of the corresponding loss of specific momentum to the ideal value of the specific momentum in the interior. The numerical value of this coefficient depends on the physicochemical properties of the fuel components, the retardation pressure of the flow, and the dimensions of the nozzle. For certain combinations of these characteristics the specific momentum loss component under consideration becomes several times greater than the sum of the losses due to friction and dissipation in the nozzle.

The theoretical determination of ξ_n is carried out by numerical integration of the system of differential equations representing chemical kinetics and gasdynamics. The results of a parametric study of the losses due to the lack of chemical equilibrium in the flow of combustion products of certain fuels may be found in an existing reference book [1].

In view of the great difficulty of the problem under consideration, attempts are generally made to derive empirical relationships generalizing the results of numerical integration. For example, in one paper [2] curves are presented for interpolating the results of these calculations by reference to the retardation pressure and the characteristic dimensions of the nozzle. We note that such curves are only valid for a narrow range of variation of the arguments.

This paper is devoted to the derivation of an approximate criterial relationship for determining the losses of specific momentum due to the lack of chemical equilibrium of the flow in a circular profiled nozzle, suitable for a variety of fuels over wide ranges of variation of the defining parameters. The basis of this investigation lies in the data published in [1] and the similarity-theory methods described in [3].

The reference book in question presents the results of calculations in which use was made of the velocity constants of the chemical reactions taken from [4]. The calculations were carried out by the method proposed

TABLE 1. Approximation Coefficients for $O_2 - H_2$ Fuel

α	$h \cdot 10^6$, m	ξ_0	m	n	s
0,6	20	0,25	0,090	0,25	0,09
0,8	220	0,25	0,110	0,28	0,20
1,0	4,5	0,33	0,090	0,20	0,25
1,2	2,0	0,38	0,085	0,20	0,20

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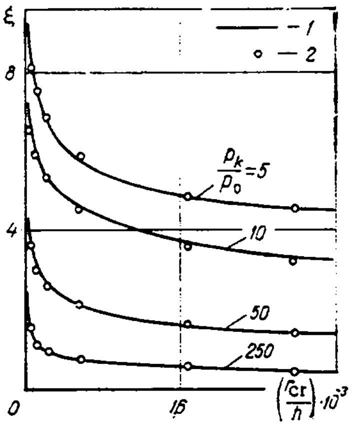

Fig. 1. Loss coefficient due to the lack of chemical equilibrium in the flow as a function of the dimensionless radius of the critical nozzle cross section for O_2-H_2 fuel at $\alpha = 1$ and $r_a/r_{cr} = 10$: 1) proposed relationship; 2) numerical integration ξ_n, %.

in [5]. The results of the numerical integration are presented in the form of curves of the function

$$\xi_n = f\left(\frac{r_a}{r_{cr}},\ r_{cr},\ p_k,\ \alpha\right). \tag{1}$$

On the basis of the differential equations and the initial conditions of the problem we may conclude that, apart from the quantities entering into Eq. (1), the list of defining parameters should include the normal pressure $p_0 = 101325\ N/m^2$ and a certain quantity h proportional to the average length of the relaxation zone. The quantity h may be regarded as an overall kinetic characteristic of the combustion products. Subsequently this quantity will be called the relaxation coefficient. This coefficient lies in the range $0 < h < \infty$.

The lower limit corresponds to equilibrium and the upper to completely nonequilibrium flow. For each specific fuel composition the relaxation coefficient may be determined from the results of a numerical integration of the problem for 6-8 values of r_a, r_{cr}, and p_k if a mathematical model representing the dependence of ξ_n on the dimensionless defining parameters is chosen in advance.

In this paper we choose a mathematical model allowing for the following boundary conditions:

1) when r_{cr}/h, $p_k/p_0 \to \infty$ and $r_a/r_{cr} \to 1$ the composition of the combustion products tends to equilibrium and hence $\xi_n \to 0$;

2) when r_{cr}/h, $p_k/p_0 \to 0$ and $r_a/r_{cr} \to \infty$ the composition of the combustion products tends to complete nonequilibrium and hence $\xi_n \to \xi_0$, where ξ_0 is the maximum value of the loss coefficient due to the lack of chemical equilibrium.

The following relationship satisfies the results of the numerical solution of the problem given in [1] and the foregoing boundary conditions:

$$\xi_n = \xi_0\,[\exp y_1 + \exp y_2 - \exp(y_1 + y_2)], \tag{2}$$

where

$$y_1 = -\left(\frac{r_{cr}}{h}\right)^m \left(\frac{p_k}{p_0}\right)^n \left(\frac{r_a}{r_{cr}} - 1\right)^{-s},$$
$$y_2 = -C\left(\frac{r_{cr}}{h}\right)^u \left(\frac{p_k}{p_0}\right)^v \left(\frac{r_a}{r_{cr}} - 1\right)^{-q}. \tag{3}$$

The quantities C, m, n, s, u, v, and q entering into this relationship are dimensionless coefficients. Analysis of the results of the calculations shows that for $r_{cr}/h < 2\cdot 10^5$, $p_k/p_0 < 500$ and $r_a/r_{cr} > 3$ we may take $u = v = 0$ and $q = s$. Equation (3) then takes the form

$$y_2 = -C\left(\frac{r_a}{r_{cr}} - 1\right)^{-s},$$

where C ≫ 1. For certain fuels we may take C = ∞, and then Eq. (2) simplifies; the terms containing y_2 vanish.

By way of example, Table 1 gives the approximation coefficients for O_2-H_2 fuel and four values of the excess coefficient of the oxidizing agent. The quantities in this table were determined by the well-known method employed for the analytical representation of empirical data [6]. In the present example C = ∞.

The results of our determination of ξ_n for oxygen–hydrogen fuel with $\alpha = 1$ and $r_a/r_{cr} = 10$ by means of Eq. (2) and also by numerical integration of the problem are presented in Fig. 1. These data, together with the results of analogous calculations carried out for $p_k = 0.5$-25 MN/m^2, $r_{cr} = (2.5$-$125)\ 10^{-3}$ m and $r_a/r_{cr} = 3$-15 in the case of fuels containing hydrogen, carbon, nitrogen, oxygen, and fluorine, show that the maximum error in the determination of losses due to the lack of chemical equilibrium in the flow through the nozzle by means of the proposed approximate equation equals ±0.002. Such an error in ξ_n introduces an error not exceeding 0.2% into the determination of the specific momentum.

NOTATION

ξ_n, loss coefficient of the specific momentum due to the lack of chemical equilibrium in the flow; r_{cr}, radius of critical nozzle cross section; r_a, radius of nozzle outlet section; p_k, flow retardation pressure; p_0, normal pressure; h, relaxation coefficient; ξ_0, m, n, s, C, dimensionless coefficients; α, excess coefficient of oxidizing agent.

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COUPLED HEAT TRANSFER BETWEEN FLUID FILM AND SOLID WALL

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A numerical algorithm is proposed for solution of the coupled problem of convective heat transfer. The method was used to study two-phase heat transfer between a solid wall and a laminarly flowing fluid film for a linear temperature profile at the outer surface of the wall. A computational formula is proposed for the dimensionless Nusselt number.

In studying heat transfer in a film flowing gravitationally along a wall, the temperature at the solid–fluid-film interface is usually assumed known and equal to a given temperature at the outer surface of the wall. This condition is satisfied in the extremely idealized case of a wall with infinitely large thermal conductivity. However, the coefficients of thermal conductivity for several polymer materials such as Teflon and vinyl are of the

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